West Virginia Mathematics

Trigonometry/Pre-calculus – Mathematics IV

Adopted 2024

Trigonometry/Pre-calculus – Mathematics IV

Building Relationships among Complex Numbers, Vectors, and Matrices

1. Perform arithmetic operations with complex numbers. TPC.BR.1
    1. Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers. M.4HSTP.1
2. Represent complex numbers and their operations on the complex plane. TPC.BR.2
    2. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number. M.4HSTP.2
    3. Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. M.4HSTP.3
    4. Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints. M.4HSTP.4
3. Represent and model with vector quantities. TPC.BR.3
    5. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes. M.4HSTP.5
    6. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. M.4HSTP.6
    7. Solve problems involving velocity and other quantities that can be represented by vectors. M.4HSTP.7
4. Perform operations on vectors. TPC.BR.4
    8. Add and subtract vectors. M.4HSTP.8
        a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. M.4HSTP.8.A
        b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. M.4HSTP.8.B
        c. Describe vector subtraction in terms of vector addition, represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. M.4HSTP.8.C
    9. Multiply a vector by a scalar. M.4HSTP.9
        a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise. M.4HSTP.9.A
        b. Compute the magnitude of a scalar multiple of a vector. M.4HSTP.9.B
5. Perform operations on matrices and use matrices in applications. TPC.BR.5
    10. Use matrices to represent and manipulate data. M.4HSTP.10

11. Multiply matrices by scalars to produce new matrices. M.4HSTP.11
12. Add, subtract and multiply matrices of appropriate dimensions. M.4HSTP.12
13. Demonstrate that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. M.4HSTP.13
14. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. M.4HSTP.14
15. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. M.4HSTP.15
16. Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. M.4HSTP.16

6. Solve systems of equations. TPC.BR.6

17. Represent a system of linear equations as a single matrix equation in a vector variable. M.4HSTP.17
18. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). M.4HSTP.18

Trigonometric and Inverse Trigonometric Functions of Real Numbers

1. Extend the domain of trigonometric functions using the unit circle. TPC.TIF.1

    19. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. M.4HSTP.19

    20. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. M.4HSTP.20

    21. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. M.4HSTP.21

    22. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. M.4HSTP.22

2. Model periodic phenomena with trigonometric functions. TPC.TIF.2

    23. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. M.4HSTP.23

    24. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. M.4HSTP.24

    25. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in terms of the context. M.4HSTP.25

    26. Solve multi-step trigonometric equations that require factoring or the use of identities. M.4HSTP.26

3. Prove and apply trigonometric identities. TPC.TIF.3

    27. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. M.4HSTP.27

    28. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. M.4HSTP.28

Analysis and Synthesis of Functions

1. Interpret functions that arise in applications in terms of a context. TPC.ASF.1

    29. Select a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its graph based on the behavior of data and context, and, where applicable, to the quantitative relationship it describes.
Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. M.4HSTP.29

2. Build a function that models a relationship between two quantities. TPC.ASF.2

    30. Write a function that describes a relationship between two quantities, including composition of functions. M.4HSTP.30

3. Analyze functions using different representations. TPC.ASF.3

    31. Graph trigonometric and rational functions expressed symbolically and show key features of the graph. M.4HSTP.31

        a. For trigonometric functions, focus on period, midline, amplitude, and phase shift. M.4HSTP.31.A

        b. For rational functions, focus on identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Analyze asymptotes and continuity informally using limits. M.4HSTP.31.B

4. Build new functions from existing functions. TPC.ASF.4

    32. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types and use transformations to model situations. M.4HSTP.32

    33. Find inverse functions. M.4HSTP.33

        a. Verify by composition that one function is the inverse of another. M.4HSTP.33.A

        b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Compute values of inverse functions from graphs and recognize the graph of an inverse function is the graph of the original function reflected about y=x. M.4HSTP.33.B

        c. Produce an invertible function from a non-invertible function by restricting the domain. M.4HSTP.33.C

    34. Use an understanding of the inverse relationship between exponents and logarithmic functions to: M.4HSTP.34

        a. Graph logarithms, M.4HSTP.34.A

b. Derive properties of logarithms, and M.4HSTP.34.B

c. Use these properties to model and solve problems and applications involving exponential and logarithmic functions. M.4HSTP.34.C

Derivations in Analytic Geometry

1. Use conic sections to solve applications. TPC.DAG.1

35. Derive the equations of a parabola, circle, ellipses, and hyperbolas using their key components. Graph the equations of these conic sections.
Key components include asymptotes, center, directrix, foci, and radius. M.4HSTP.35

36. Solve problems and applications that model conic sections. M.4HSTP.36

Series and Informal Limits

1. Use sigma notations to evaluate finite sums. TPC.S.1

37. Develop sigma notation and use it to write series in equivalent form. M.4HSTP.37

38. Apply the method of mathematical induction to prove summation formulas. For example, verify the sum of squares formula. M.4HSTP.38

2. Extend geometric series to infinite geometric series. TPC.S.2

39. Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Apply infinite geometric series models intuitively. M.4HSTP.39