Minnesota Mathematics

Grades 9-11

Data Analysis

1 Data Sciences: Identify, formulate and investigate statistical questions by collecting data considering cultural perspectives, analyzing and interpreting data and communicating the results.

1 Formulate statistical investigative questions and pose hypotheses. These include questions about variation or the differences between groups, associations between quantitative and categorical variables or pairing together multiple analyses. (MP1, MP3) $ µ ☼ ✣ 9.1.1.1

2 Explain how choices concerning data collection methods can affect the quality, size, speed, accessibility and cost of the data. (MP3) $ ☼ 9.1.1.2

3 Analyze issues of bias by considering data collection methods and cultural perspectives. (MP3) ☼ 9.1.1.3

4 Explain the purposes of and differences among sample surveys, experiments and observational studies. Explain whether randomization in each allows for conclusions of causation and/or generalization of a population. (MP3) ☼ 9.1.1.4

5 Analyze and explain when arguments based on data confuse correlation and causation. (MP3) 9.1.1.5

6 Compute using technology or estimate the correlation coefficient of a linear model. Interpret the linear model in the context of the data. (MP5, MP6) $ 9.1.1.6

7 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Identify data sets for which such a procedure is not appropriate. Tools can include calculators, spreadsheets, apps or tables. (MP5, MP6) $ ☼ 9.1.1.7

8 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples or simulated samples of the same size to gauge the variation in estimates or predictions. (MP1, MP4) # ☼ 9.1.1.8

9 Use statistics appropriate to the shape of the data distribution to compare the center and spread of two or more data sets. (MP4) 9.1.1.9

10 Create and analyze data displays, including scatter plots, histograms and boxplots using technology. (MP1) ☼ 9.1.1.10

11 Identify, create and compare statistical models with linear and exponential functions, including linear regression. Assess the reasonableness of model fit using residuals and correlation coefficients. (MP4) # µ 9.1.1.11

12 Examine and discuss competing explanations for data trends observed such as confounding variables. Respond to competing arguments or interpretations of the data of different community groups, paying careful attention to what conclusions the data supports. (MP3, MP6) $ ☼ 9.1.1.12

13 Analyze and interpret data using various measures, such as difference in shapes, center and spread to draw conclusions, identify trends and describe relationships, accounting for possible effects of extreme data points (outliers). (MP1) $ 9.1.1.13

14 Evaluate reports based on published data by identifying the source of the data, the design of the study and the way the data are analyzed and displayed. (MP3) $

☼ 9.1.1.14

15 Identify and explain misleading uses of data along with how to use spreadsheets, tables or graphing technology to recognize and analyze distortions in data displays. Use interactive data visualizations to support and influence different points of view. (MP3) $ # 9.1.1.15

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2 **Chance and Uncertainty: Apply and explain the concepts of probability to interpret data, generate questions, predict and make informed decisions to solve problems and communicate ideas.**

1 Select and apply counting procedures such as the multiplication and addition principles, permutations, combinations and tree diagrams to determine the elements and size of a sample space. (MP1, MP7)

2 Describe events as subsets of a sample space using characteristics of the outcomes. Use Venn diagrams, tables or lists to depict events and compute event sizes. Create representations to find unions, intersections and complements of events. (MP1) #

3 Apply probability concepts, such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve situations. (MP4) # ☼

4 Calculate experimental probabilities by repeatedly performing simulations or experiments involving a probability model and by using relative frequencies of outcomes. Recognize that as the number of trials increases, the closer the experimental probability approaches the theoretical probability. (MP4)

5 Construct and interpret two-way frequency tables when two categories are associated with each object being classified. Use the two-way table to decide if events are independent and to calculate conditional probabilities. (MP4, MP5) ☼

6 Compare the recursion to create the nth row of Pascal's triangle, the recursion to expand $(xx + yy)nn$, the recursion to generate all sequences of heads and tails on n coin flips and the recursion to generate all subsets of a set with n elements. Establish connections among the various representations. (MP8)

7 Calculate and make decisions based on expected values using theoretical or experimental probabilities in scenarios, including finance or games of chance. (MP1, MP6) $ μ ☼

8 Calculate and make decisions to obtain a desired outcome (maximum or minimum value or maximum or minimum probability) based on expected value and theoretical or experimental probabilities in scenarios such as games of chance or finance. (MP6, MP7) $ ☼

Spatial Reasoning

3 **Measurement: Investigate measurement using a variety of tools, units, systems, processes and techniques in various cultures. Explain and reason with attributes, estimations and formulas to communicate measurement(s) and relationships effectively. Justify decisions and consider the reasonableness of the measurement.**

1 Apply the Pythagorean Theorem and distance formula to compute perimeters of polygons and areas of right triangles and rectangles on coordinate planes. (MP1) ☼ 9.2.3.1

2 Apply properties of 45°-45°-90° and 30°-60°-90° triangles to solve situations and logically justify results. (MP1, MP4) ☼ 9.2.3.2

3 Use trigonometric ratios and the Pythagorean Theorem and its converse to solve contextual situations, including right triangles or polygons that can be decomposed into triangles. (MP4, MP7) ☼ 9.2.3.3

4 Develop, justify and use decomposition to determine the formula for surface area and volume of various three-dimensional figures. (MP7, MP8) ✜ # ☼ 9.2.3.4

5 Use the formulas for surface area and volume of various three-dimensional figures to solve multi- step contextual situations. (MP4, MP5) ✜ ☼ 9.2.3.5

6 Use units of measure and dimensional analysis to solve multi-step situations. Interpret units consistently in formulas. Interpret the scale and the origin in graphs and data displays. (MP1, MP4) $ ☼ ✜ 9.2.3.6

7 Compute the unit cost and total cost to cover different areas and volumes in applications, such as painting, carpeting and gardening. Optimize costs for various models, including linear and quadratic. Use technology, including spreadsheets. (MP4, MP5) $ µ ☼ 9.2.3.7

8 Use similarity to determine the side ratios in right triangles as properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (MP8) 9.2.3.8

9 Apply dilations of scale factor k on length, area and volume and recognize the effect is multiplication by kk, $kk2$, $kk3$, respectively. (MP2) µ ☼ 9.2.3.9

4 **Geometry: Analyze characteristics of geometric shapes to make mathematical arguments and justifications about geometric relationships. Use visualization and geometric modeling to compare, solve problems and communicate ideas.**

1 Apply properties of angles, including corresponding, exterior, interior, vertical, complementary and supplementary angles, to solve situations and logically justify results. (MP1, MP3) 9.2.4.1

2 Apply properties of equilateral, isosceles and scalene triangles to solve situations and logically justify results. (MP1, MP3) ✣ ☼ 9.2.4.2

3 Apply properties such as interior and exterior angles and parallel and perpendicular sides to define, classify and solve situations involving regular and irregular polygons. (MP1) ✣ ☼ 9.2.4.3

4 Apply properties of congruent figures to solve situations and logically justify results. (MP1, MP3) ✣ ☼ 9.2.4.4

5 Accurately interpret and use words and phrases such as “if ... then,” “if and only if,” “all” and “not.” Recognize the logical relationships between an “if ... then” statement and its inverse, converse and contrapositive. (MP6, MP7) ✣ # ☼ 9.2.4.5

6 Analyze the validity of a logical argument and give counterexamples to disprove a statement. (MP1, MP3) ✣ # ☼ 9.2.4.6

7 Construct logical arguments using axioms, definitions, theorems and postulates that clearly justify the reasoning. (MP3, MP6) # 9.2.4.7

8 Identify, describe and solve using relationships among inscribed angles, circumscribed angles, radii and chords in circles. (MP5) ✣ ☼ 9.2.4.8

9 Describe and solve using the relationships of angles formed outside the circle for both secants and tangent lines. (MP7) ☼ 9.2.4.9

10 Apply properties of similar figures to solve situations and logically justify results. (MP3) ✣ 9.2.4.10

11 Show that two triangles are similar using the AA, SAS and SSS similarity criteria. (MP7) 9.2.4.11

12 Make geometric constructions with a variety of tools, including dynamic geometric technology, to examine theorems, make and test conjectures, represent transformations and develop mathematical reasoning skills in multi-step situations. (MP5) 9.2.4.12

13 Create and compare rigid and non-rigid transformations, and connect the preimage and image to congruence. (MP2) 9.2.4.13

14 Describe the steps used to show various sequences of transformations, applying reflections, rotations, translations and/or dilations of geometric figures that map one figure onto the other. (MP8) # ✣ 9.2.4.14

15 Apply geometric methods to solve design situations, including designing an object or structure to satisfy physical constraints or optimize resources. (MP4) $ µ 9.2.4.15

16 Apply concepts of density based on area and volume in modeling contextual situations, including population density, BTUs per cubic foot and data storage.

(MP4) 9.2.4.16

Patterns and Relationships

5 **Number Relationships: Describe/Interpret and use quantities, relationships between and representations of quantities and number systems. Describe and relate operations. Use strategies and procedures accurately, efficiently and flexibly. Assess the reasonableness of the results.**

1 Add, subtract, multiply and divide numbers in scientific notation. (MP6, MP7) ✜ ☼ 9.3.5.1

2 Compare the definition of rational exponents and properties of radicals. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents. (MP6) 9.3.5.2

3 Apply knowledge of number systems extending from whole numbers to integers, from integers to rational numbers, from rational numbers to real numbers and from real numbers to complex numbers to solve equations. (MP2, MP7) 9.3.5.3

4 Use matrices to represent and manipulate data and interpret the results in context. Add, subtract and multiply matrices of appropriate dimensions. Multiply matrices by scalars. (MP4, MP5) # 9.3.5.4

5 Estimate and verify the cost of an item, including multiple discounts and taxes. Show an understanding of the order of operations. (MP6, MP8) $ 9.3.5.5

6 Assess the reasonableness of a solution in its given context, including financial literacy applications. Compare the solution to appropriate graphical or numerical estimates. Interpret a solution in the original context. (MP3) $ ☼ 9.3.5.6

7 Use the structure of an expression, equation and/or formula to create an equivalent form that is more helpful given the situation. Rearrange formulas to highlight a quantity of interest, using the same reasoning in solving equations. (MP6) $ 9.3.5.7

8 Use the structure of an expression to write it in multiple ways. (MP7) 9.3.5.8

9 Find and evaluate the composition of multiple functions. (MP7) 9.3.5.9

10 Compute the time it takes to pay off a loan given the interest rate, loan amount and monthly payment. Determine the change in cost by the variation in loan amount, fixed or variable interest rates and payment amounts. Use technology, including spreadsheets. (MP1, MP5) ✜ $ ☼ 9.3.5.10

11 Compute the total payments to pay off a loan given the interest rate, loan amount and monthly payment. Compare the total payments for various monthly payments and various interest rates. Use technology, including spreadsheets. (MP1, MP5) ✜ $ ☼ 9.3.5.11

12 Compare different types of retirement plans, considering after tax or pretax contributions, employer match contributions, the benefit of starting contributions early and the consequences of use prior to retirement. (MP1) $ 9.3.5.12

6 **Equivalence and Relational Thinking: Use concepts and properties of equivalence and relational thinking to represent and compare numerical expressions, proportional relationships, algebraic expressions and equations.**

1 Add, subtract and multiply polynomials. (MP1, MP7) 9.3.6.1

2 Reason abstractly to compare general forms of quadratics, including vertex form, general form, factored form and the graph. Develop procedures to convert from one form to another. (MP2, MP7) µ 9.3.6.2

3 Choose and produce an equivalent form of a quadratic function, using symbolic and graphical methods, to identify the vertex, line of symmetry and intercepts of the parabola. (MP8) 9.3.6.3

4 Factor common monomial factors from polynomials, quadratic polynomials and the difference of two squares. (MP7, MP8) 9.3.6.4

5 Solve quadratic equations by appropriate methods using factoring, completing the square, graphing or the quadratic formula. Find non-real complex roots when they exist. (MP2, MP7) 9.3.6.5

6 Compare the equation of a circle, the Pythagorean Theorem and the Distance Formula. Complete the square to find the center and radius of a circle given an equation. (MP2, MP8) 9.3.6.6

7 Solve situations involving relationships which are inversely proportional in various contexts, including rates. (MP4) ✜ $ ☼ 9.3.6.7

8 Apply the properties of rational exponents and radicals to generate equivalent algebraic expressions. (MP2, MP7) 9.3.6.8

7 **Patterns and Relationships: Represent and connect mathematical patterns and relationships using verbal descriptions, generalizations, tables and graphs. Use representations to generate questions, make predictions and solve mathematical problems.**

1 Represent and solve situations in various contexts, including financial literacy, using systems of linear equations, systems of linear inequalities and exponential and quadratic functions. (MP4) $ # ☼ 9.3.7.1

2 Translate between graphs of quadratic, exponential and other functions (including absolute value, rational and polynomial), tables and symbolic representations. Sketch graphs and use graphing technology to graph functions. (MP5) $ 9.3.7.2

3 Determine how vertical/ horizontal reflecting, translating and scaling affect the symbolic and graphical forms of a function. Use graphing technology to examine transformations. (MP3) 9.3.7.3

4 Express the terms in an arithmetic or geometric sequence recursively and by giving an explicit formula. (MP8) # 9.3.7.4

5 Express recursive patterns using recursive formulas. Calculate sequences defined by recursive formulas. (MP8) $ 9.3.7.5

6 Find the domain and range of functions defined symbolically, graphically or in a context, including piecewise and step functions. Express solutions and recognize that some answers obtained may not be valid, including cases where the function inputs are discrete instead of continuous. (MP4) ✜ $ ☼ 9.3.7.6

7 Describe the graph of a function using key features such as intercepts, maxima/minima, intervals of increase and decrease and end behavior. Draw conclusions from graphs of functions and other relations. (MP3) $ ☼ 9.3.7.7

8 Define the compounding of interest n times per year according to a recursive formula. Compare the recursive definition of interest to the recursive definition of a geometric sequence $tt(nn) = rr(tt(nn - 1))$. Compare the interest formula $AA = nnnn$ $\blacklozenge 1 + rr\blacklozenge$ to the general form of an exponential nn function $yy = aa(bb)xx$. Explain the purpose of each part of the interest formula. (MP4, MP5) $ # µ 9.3.7.8

9 Find the inverse of a given function and justify the results using tables, graphs or algebra. (MP4, MP6) 9.3.7.9

10 Use the concept of a function as a connection between inputs and outputs to find function values and use function notation. (MP2) 9.3.7.10