

# Multivariable Calculus

## Mathematical Practices

**0 Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.** *MVC.MP*

**0.1** Make sense of problems and persevere in solving them. *MVC.MP.1*

**0.2** Reason abstractly and quantitatively. *MVC.MP.2*

**0.3** Construct viable arguments and critique the reasoning of others. *MVC.MP.3*

**0.4** Model with mathematics. *MVC.MP.4*

**0.5** Use appropriate tools strategically. *MVC.MP.5*

**0.6** Attend to precision. *MVC.MP.6*

**0.7** Look for and make use of structure. *MVC.MP.7*

**0.8** Look for and express regularity in repeated reasoning. *MVC.MP.8*

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## Mathematical Modeling

**1 Apply mathematics to real-life situations; model real-life phenomena using mathematics.** *MVC.MM.1*

**1.1** Explain contextual, mathematical problems using a mathematical model. *MVC.MM.1.1*

**1.2** Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts. *MVC.MM.1.2*

**1.3** Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation. *MVC.MM.1.3*

**1.4** Use various mathematical representations and structures with this information to represent and solve real-life problems. *MVC.MM.1.4*

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## Patterning & Algebraic Reasoning

### 2 Express spatial and functional relationships with vectors, functions, and analytic geometry in three dimensions, and use these relationships to solve contextual, mathematical problems. [MVC.PAR.2](#)

- 2.1 Represent equations of lines in space using vectors. [MVC.PAR.2.1](#)
  - 2.2 Express the analytic geometry of three dimensions in terms of the dot product and cross product of vectors. [MVC.PAR.2.2](#)
  - 2.3 Use a linear system of equations to determine whether two planes intersect in a single point or a line, or whether they do not intersect at all. [MVC.PAR.2.3](#)
  - 2.4 Evaluate functions of two independent variables at a point in the plane. [MVC.PAR.2.4](#)
  - 2.5 Graph the level curves of functions of two independent variables. [MVC.PAR.2.5](#)
  - 2.6 Investigate the continuity of functions of two independent variables in terms of the limits of such functions as  $(x, y)$  approaches a given point in the plane. [MVC.PAR.2.6](#)
  - 2.7 Determine points or regions of discontinuity of functions of two independent variables. [MVC.PAR.2.7](#)
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## Abstract & Quantitative Reasoning

### 3 Define, describe, and represent the differentiation of functions of two independent variables and differential vectors to solve contextual, mathematical problems and to explain real-life phenomena. [MVC.AQR.3](#)

- 3.1 Approximate the partial derivatives at a point of a function defined by a table of data. [MVC.AQR.3.1](#)
- 3.2 Find expressions for the first and second partial derivatives of a function. [MVC.AQR.3.2](#)
- 3.3 Use the total differential to approximate mathematical models. [MVC.AQR.3.3](#)
- 3.4 Represent the partial derivatives of a system of two functions in two variables using the Jacobian. [MVC.AQR.3.4](#)
- 3.5 Find the partial derivatives of the composition of functions using the general chain rule. [MVC.AQR.3](#)
- 3.6 Apply partial differentiation to problems of optimization, including problems requiring the use of the Lagrange multiplier. [MVC.AQR.3.5](#)
- 3.7 Find the family of solutions and the envelope of the family of solutions to differential equations, including Clairaut equations. [MVC.AQR.3.6](#)
- 3.8 Define and apply the gradient, the divergence, and the curl in terms of differential vector operations. [MVC.AQR.3.7](#)

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**4 Interpret integrals of functions of two independent variables and of vector functions to solve contextual, mathematical problems and to explain real-life phenomena.** *MVC.AQR.4*

- 4.1 Integrate functions of the form  $z = f(x, y)$  or  $w = f(x, y, z)$  through various techniques. *MVC.AQR.4.1*
- 4.2 Use, evaluate, and interpret double and triple integrals in terms of volume and mass. *MVC.AQR.4.2*
- 4.3 Represent and evaluate integrals of vector functions as double and triple integrals. *MVC.AQR.4.3*
- 4.4 Apply line and surface integral to functions representing real-world phenomena. *MVC.AQR.4.4*
- 4.5 Solve first-order exact differential equations. *MVC.AQR.4.5*
- 4.6 Use Green's Theorem to evaluate line integrals in the plane; use Stokes' Theorem to evaluate line integrals in space. *MVC.AQR.4.6*
- 4.7 Determine whether a line integral is independent of path and use line integrals in context. *MVC.AQR.4.7*
- 4.8 Use Gauss' Divergence Theorem to evaluate surface integrals. *MVC.AQR.4.8*
- 4.9 Define and apply the gradient, the divergence, and the curl in terms of integrals of vector functions. *MVC.AQR.4.9*