

# Grades 9-12: Algebra I With Probability

Adopted 2019

## Algebra I With Probability

### Number and Quantity

- A. Together, irrational numbers and rational numbers complete the real number system, representing all points on the number line, while there exist numbers beyond the real numbers called complex numbers. [A1P.NQ.A](#)
  - 1. Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents. [A1P.NQ.A.1](#)
  - 2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [A1P.NQ.A.2](#)
  - 3. Define the imaginary number  $i$  such that  $i^2 = -1$ . [A1P.NQ.A.3](#)

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## Algebra and Functions

- A. Expressions can be rewritten in equivalent forms by using algebraic properties, including properties of addition, multiplication, and exponentiation, to make different characteristics or features visible. **A1P.AF.A**
4. Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. **A1P.AF.A.4**
  5. Use the structure of an expression to identify ways to rewrite it. **A1P.AF.A.5**
  6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A1P.AF.A.6**
    - a. Factor quadratic expressions with leading coefficients of one, and use the factored form to reveal the zeros of the function it defines. **A1P.AF.A.6.A**
    - b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. **A1P.AF.A.6.B**
    - c. Use the properties of exponents to transform expressions for exponential functions. **A1P.AF.A.6.C**
  7. Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication. **A1P.AF.A.7**
- B. Finding solutions to an equation, inequality, or system of equations or inequalities requires the checking of candidate solutions, whether generated analytically or graphically, to ensure that solutions are found and that those found are not extraneous. **A1P.AF.B**
8. Explain why extraneous solutions to an equation involving absolute values may arise and how to check to be sure that a candidate solution satisfies an equation. **A1P.AF.B.8**
- C. The structure of an equation or inequality (including, but not limited to, one-variable linear and quadratic equations, inequalities, and systems of linear equations in two variables) can be purposefully analyzed (with and without technology) to determine an efficient strategy to find a solution, if one exists, and then to justify the solution. **A1P.AF.C**
9. Select an appropriate method to solve a quadratic equation in one variable. **A1P.AF.C.9**
    - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Explain how the quadratic formula is derived from this form. **A1P.AF.C.9.A**
    - b. Solve quadratic equations by inspection (such as  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real. **A1P.AF.C.9.B**

10. Select an appropriate method to solve a system of two linear equations in two variables. **A1P.AF.C.10**
  - a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. **A1P.AF.C.10.A**
  - b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods. **A1P.AF.C.10.B**
- D. Expressions, equations, and inequalities can be used to analyze and make predictions, both within mathematics and as mathematics is applied in different contexts – in particular, contexts that arise in relation to linear, quadratic, and exponential situations. **A1P.AF.D**
  11. Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions. **A1P.AF.D.11**
  12. Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. **A1P.AF.D.12**
  13. Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and linear piecewise functions. **A1P.AF.D.13**
- E. Functions shift the emphasis from a point-by-point relationship between two variables (input/output) to considering an entire set of ordered pairs (where each first element is paired with exactly one second element) as an entity with its own features and characteristics. **A1P.AF.E**
  14. Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. **A1P.AF.E.14**
  15. Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. **A1P.AF.E.15**
    - a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **A1P.AF.E.15.A**
    - b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions. **A1P.AF.E.15.B**
  16. Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a

function. Explain that a function  $y = f(x)$  is a special kind of relation defined by the equation  $y = f(x)$ . **A1P.AF.E.16**

**17.** Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions. **A1P.AF.E.17**

**a.** Use arithmetic operations to combine different types of standard functions to write and evaluate functions. **A1P.AF.E.17.A**

**b.** Use function composition to combine different types of standard functions to write and evaluate functions. **A1P.AF.E.17.B**

**F.** Graphs can be used to obtain exact or approximate solutions of equations, inequalities, and systems of equations and inequalities – including systems of linear equations in two variables and systems of linear and quadratic equations (given or obtained by using technology). **A1P.AF.F**

**18.** Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate. **A1P.AF.F.18**

**19.** Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ . **A1P.AF.F.19**

**a.** Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where  $f(x)$  is a linear, quadratic, exponential, or absolute value function and  $g(x)$  is constant or linear. **A1P.AF.F.19.A**

**20.** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate. **A1P.AF.F.20**

**G.** Functions can be described by using a variety of representations: mapping diagrams, function notation (e.g.,  $f(x) = x^2$ ), recursive definitions, tables, and graphs. **A1P.AF.G**

**21.** Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise. **A1P.AF.G.21**

**22.** Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. **A1P.AF.G.22**

**a.** Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. **A1P.AF.G.22.A**

**H.** Functions that are members of the same family have distinguishing attributes (structure) common to all functions within that family. **A1P.AF.H**

23. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k \cdot f(x)$ ,  $f(f(x))$ , for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Limit to linear, quadratic, exponential, absolute value, and linear piecewise functions. [A1P.AF.H.23](#)
24. Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. [A1P.AF.H.24](#)
- Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. [A1P.AF.H.24.A](#)
  - Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. [A1P.AF.H.24.B](#)
  - Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [A1P.AF.H.24.C](#)
25. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [A1P.AF.H.25](#)
26. Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. [A1P.AF.H.26](#)
27. Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form  $mx + b$ , to exponential functions, written in the form  $ab^{x/c}$ . Functions can be represented graphically and key features of the graphs, including zeros, intercepts, and, when relevant, rate of change and maximum/minimum values, can be associated with and interpreted in terms of the equivalent symbolic representation. [A1P.AF.H.27](#)
28. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and linear piecewise functions. [A1P.AF.H.28](#)
29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions. [A1P.AF.H.29](#)
30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. [A1P.AF.H.30](#)
- Graph linear and quadratic functions and show intercepts, maxima, and minima. [A1P.AF.H.30.A](#)
  - Graph piecewise-defined functions, including step functions and absolute value functions. [A1P.AF.H.30.B](#)

- c. Graph exponential functions, showing intercepts and end behavior. **A1P.AF.H.30.C**
- l. Functions model a wide variety of real situations and can help students understand the processes of making and changing assumptions, assigning variables, and finding solutions to contextual problems. **A1P.AF.I**
- 31.** Use the mathematical modeling cycle to solve real-world problems involving linear, quadratic, exponential, absolute value, and linear piecewise functions. **A1P.AF.I.31**

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## Data Analysis, Statistics, and Probability

- A. Mathematical and statistical reasoning about data can be used to evaluate conclusions and assess risks. **A1P.DSP.A**
  - A32.** Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. **A1P.DSP.A32**
- B. Making and defending informed, data-based decisions is a characteristic of a quantitatively literate person. **A1P.DSP.B**
  - 33.** Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. **A1P.DSP.B.33**
- C. Data arise from a context and come in two types: quantitative (continuous or discrete) and categorical. Technology can be used to "clean" and organize data, including very large data sets, into a useful and manageable structure—a first step in any analysis of data. **A1P.DSP.C**
  - 34.** Distinguish between quantitative and categorical data and between the techniques that may be used for analyzing data of these two types. **A1P.DSP.C.34**
- D. The association between two categorical variables is typically represented by using two-way tables and segmented bar graphs. **A1P.DSP.D**
  - 35.** Analyze the possible association between two categorical variables. **A1P.DSP.D.35**
    - a.** Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. **A1P.DSP.D.35.A**
    - b.** Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). **A1P.DSP.D.35.B**
    - c.** Identify possible associations and trends in categorical data. **A1P.DSP.D.35.C**
- E. Data analysis techniques can be used to develop models of contextual situations and to generate and evaluate possible solutions to real problems involving those contexts. **A1P.DSP.E**
  - 36.** Generate a two-way categorical table in order to find and evaluate solutions to real-world problems. **A1P.DSP.E.36**
    - a.** Aggregate data from several groups to find an overall association between two categorical variables. **A1P.DSP.E.36.A**
    - b.** Recognize and explore situations where the association between two categorical variables is reversed when a third variable is considered (Simpson's Paradox). **A1P.DSP.E.36.B**
- F. Two events are independent if the occurrence of one event does not affect the probability of the other event. Determining whether two events are independent can be used for finding and understanding probabilities. **A1P.DSP.F**

37. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [A1P.DSP.F.37](#)
38. Explain whether two events, A and B, are independent, using two-way tables or tree diagrams. [A1P.DSP.F.38](#)
- G. Conditional probabilities – that is, those probabilities that are "conditioned" by some known information – can be computed from data organized in contingency tables. Conditions or assumptions may affect the computation of a probability. [A1P.DSP.G](#)
  39. Compute the conditional probability of event A given event B, using two-way tables or tree diagrams. [A1P.DSP.G.39](#)
  40. Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. [A1P.DSP.G.40](#)
  41. Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. [A1P.DSP.G.41](#)